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ABSTRACT

After noting that research and development reports indicate that there are strong psychological arguments for approaching algebraic concepts through the use of computer programming, the work of the Nottingham Programming in Mathematics Project with pupils aged 11-13 is briefly described. What is meant by understanding of a concept and how pupils view particular concepts are discussed. Specific mathematics problems are included, as well as data from a comparison of groups using or not using computers. (MNS)

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Computer Programming and Algebra

The Nottingham Programming in Mathematics Project

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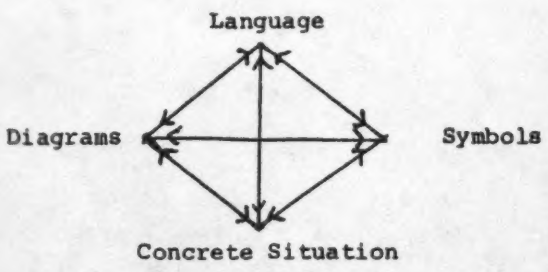
Many teachers who have done some programming in BASIC with their pupils have instinctively felt that it helps them to appreciate algebraic concepts. There is now a background of research and development which indicates that there are very strong physchological arguments for approaching such concepts through the use of computer programming. It is also becoming very evident that if every child learnt how to program in BASIC as part of his mathematics curriculum then not only methods of approach but actual content could change in a way that would make large areas more meaningful and approachable to the majority of pupils who find abstract concepts difficult. In order to appreciate the possible power of this approach it is necessary to be aware of the main thrust of the physchological arguments and how these relate to the failure of many present methods. Work has been done with 11-13 year olds which follows on from these arguments, this together with results of testing will be described.

It is easy for any teacher of mathematics to list concepts that many pupils find difficult:- place value, operations with fractions, directed numbers, ratio etc. and high on this list will appear algebraic concepts, variables, operations on variables, using functions and worst of all "two way translation", Algebra<---->English, which seems to prove impossible for all but the brightest pupils In an attempt to find reasons for these difficulties the way that 11-16 year old's view such concepts has been the subject of recent research in the U.K. (Ref.1). For the purposes of this paper two inter-related physchological factors will be considered which are:-

(a) what do we mean by understanding of a concept, and
(b) how do pupils view particular concepts.

This discussion pre-supposes that anybody involved with learning mathematics believes they understand some parts of it although other parts they may just deal with by rule manipulation. A view of the meaning of such understanding can be illustrated by the diagram below:-



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From a personal view understanding then exists if the learner can inter-relate freely between these four aspects, moreover all four

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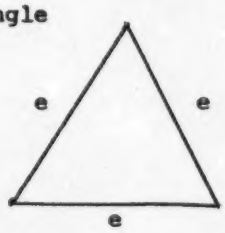
aspects are necessary so that if somebody thinks they understand a concept then they have to possess some concrete/visual image of it, unfortunately this image can often be wrong or not encompass the full realisation of the concept. Rule manipulation can fit on this model as an inter-relation between symbols and language only. The dangers of such rule manipulation are well known and obvious but the danger inherent in the existence of a naive and unhelpful image of a concept is only just coming to be realised, in particular the block it so often forms to future learning and understanding. A prime example of this is in algebra where letters used in expressions "naturally" have the image of labels or names of objects.

As an example, for the triangle



The perimeter is thought of as three lines called 'e' or three 'e' s or 3e, similarly 3a+2b is thought of as 3a's and 2b's where a and b are something solid and tangible. These images while natural do not allow of extension, problems like the simplification of 3a+b-4a arise quickly and do not fit the image, and writing correct linear algebraic expressions from statements in words becomes impossible because the correct image of a letter as a variable is not available. The necessary correct image for 3e is 3 multiplied by an unknown member of a set of numbers. Giving this as a definition will certainly not help. Finding suitable images, accessible to pupils at various stages of their development should be the essence of mathematics teaching. In this particular instance computer programming in BASIC can provide a suitable image and also lends itself very well to further developments.

Pupils involved in the Nottingham Programming in Mathematics Project learn programming at the age of 11 before they do any algebra. Their work is carefully structured and we have found that it is accessible to all ability levels. In order to see how the programming activity relates to the model of understanding given above we will start with a problem they would tackle after about four or five lessons which would have included about three hours "hands on" activity. The problem is:-

"Count how many times your heart beats in one minute, store this number in the computer, and use it to make the computer calculate how many times it beats in 1 hour, 1 day, 1 week, 1 year, and since you were born. Stick your printout into your book and write about your answers - in particular how accurate are they?"

A typical solution program is:-

```
10 LET X=78
20 LET H=X*60
30 LET D=H*24
40 LET W=D*7
50 LET Y=D*356
60 LET T=11*Y+15*W+3*D
70 PRINT H,D,W,Y,T
```

The order of letters and numbers in the multiplications reflects the way that they are expressed in English. A common comment in their writing is that the final calculation does not take account of leap years, although many also know about the variability of heart beats. In terms of images and understanding, the image that the pupil is given for the use of letters is as labels of stores whose contents can vary, a 'picture' like:-

Label	X
Contents	78

Operating on letters even with a mixture of multiplication and addition arises naturally, with the one way translation from English to 'algebra' also causing no problem. In terms of the diagram for understanding it is the image of a variable we need for further development although it does at some stage need relating back to standard algebra. In the Nottingham Project about three hundred pupils have done work of this kind so far. Some testing has been done which has consisted of giving part of the test for the algebra section of the research which lead to the results published in Children's Understanding Mathematics 11-16 (Ref.1). This was done both with pre and post tests and by comparison with parallel groups who had not used the computer. It was not possible to control the experiment closely as the work formed part of their day to day mathematical activities, but some results are given in order to provide evidence of the possibilities.

The following table provides a comparison at the end of the first year, all the pupils were from parallel mixed ability groups - the total mark was 40.

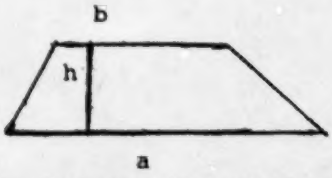
	No.of Pupils	Mean Mark	Standard Deviation
Using computer	54	13.7	5.2
Not using computer	42	9.0	4.4

Using the difference of means test for large samples gives a Z-score of 3.9 which shows that the null hypothesis that the group means are the same has to be rejected and the alternative hypothesis that the computer classes have a higher mean accepted. Other results on specific questions gave the same pattern and showed that pupils using the computer performed better than the 'average' as given in Children's

Understanding Maths 11-16 . This difference was most marked on particular questions which related closely to the way they had used letters in programming, again reinforcing the necessity of a useful image. Further details of the results of this testing are in Report No. 2 of the Project (Ref.2). It has been difficult to carry on testing in the way we started because the schools involved not surprisingly have wanted all their pupils to use computers in their Mathematics lessons.

Developments which follow from pupils being able to program in BASIC and use computers freely in mathematics classes are equally interesting. Perhaps the major feature is that the use of computer programming makes it possible for pupils with a low level of algebraic knowledge to solve problems which involve algebra either as a starting point or within the problem. As an example, a problem set to the pupils in the Project is:-

"The formula for the area of a trapezium is Area = $\frac{h(a+b)}{2}$



Write a program that enables you to input values of a, b, and h and output the area of the trapezium. Use this program to find three separate values of a, b, and h which give an area of about 5.

Draw these trapeziums on cm^2 paper and check your results.

Take another number for the area, find one set of values for a, b, and h and check the result by drawing."

You will see that this problem has many facets. It takes a less able pupil about two hours to complete, but its major virtue is that it uses algebra in a way that makes such use accessible and realistic to nearly all ability levels. There are many more possibilities using not only BASIC but also other computer facilities, with LOGO being the most notable example. These possibilities exist in virtually all branches of mathematics and a major curriculum development project is needed in order to investigate how the use of computers will transform pupil understanding and the teaching of mathematics for the pupils of the future.

References

1.Childrens Understanding of Mathematics 11-16
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2. Report No.2 The Nottingham Programming In Mathematics Project
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Trent Polytechnic 1981. (Send £1 including p.and p.)